4.1/11 $f(x) = x^2 + 8x + 1$

a) Q: is this f^n 1-1? No - it is parabola

Q: can we get that same answer looking at $f'(x)$? yes

$f'(x) = 2x + 8$

$2x + 8 = 0$

$x = -4$

$- - - \overset{0}{|} + + +$
-4

$y = x^2 + 8x + 1$

$y^{-1} = ?$

$y = x^2 + 8x + 1$

$y = (x^2 + 8x + 16) + 1 - 16$

$(x+a)(x+a)$

$x^2 + 2ax + a^2$

$y = (x+4)^2 - 15$ so ....

to find inverse switch x & y .... $x = (y+4)^2 - 15$

$x + 15 = (y+4)^2$

$\pm\sqrt{x+15} = y + 4$

4.1/11b

$$y = 2x^5 + x^3 + 3x + 2$$

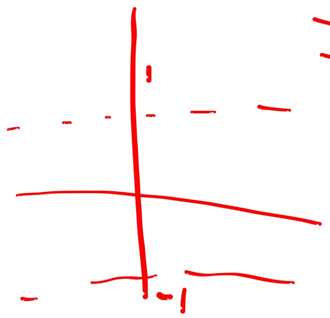
$$y' = \underbrace{10x^4 + 3x^2 + 3}_{\geq 3}$$ sign chart

INCREASING

11c) $y = 2x + \sin x$

$$y' = \underbrace{2 + \cos x \geq 1}$$

$= 0$ when $\cos x = -2$



4.1/19

$y = \sqrt[3]{2x-1}$

$(x)^3 = \left(\sqrt[3]{2y-1}\right)^3$

$x^3 = (2y-1)$

$+1 \qquad +1$

$x^3+1 = 2y$

$\frac{x^3+1}{2} = y$

Preliminary.....

$f(x)$ has a domain Dom
and a range Ran

$f^{-1}(x)$ has a domain Ran
and a range Dom

4.1/23)

$y=\frac{5}{2}-x$

$x=\frac{5}{2}-y$

$y=\frac{5}{2}-x$

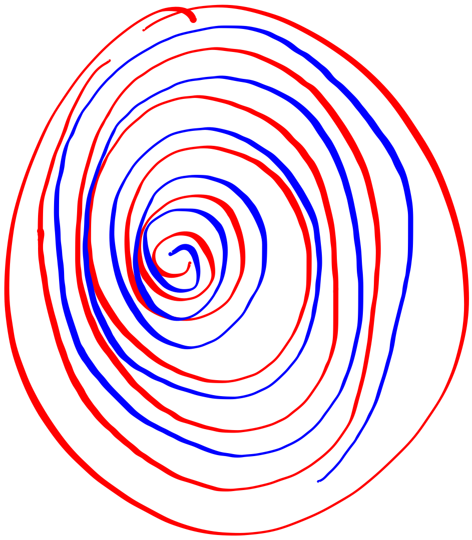
$$f(x)=\begin{cases}\frac{5}{2}-x & ; x<2 \\ \frac{1}{x} & ; x\geq 2\end{cases}$$

$y=\frac{1}{x}$

$x=\frac{1}{y}$

$y=\frac{1}{x}$

$$f^{-1}(x)=\begin{cases}\frac{5}{2}-x & ; x>\frac{1}{2} \\ \frac{1}{x} & ; x\leq\frac{1}{2}\end{cases}$$



4.1/45

$$f(x) = 5x^3 + x - 7 = y$$

$$f'(x) = 15x^2 + 1$$

(5) $$\frac{dy}{dx} = \frac{1}{dx/dy}$$

$$\frac{dy}{dx} = \frac{1}{15y^2+1}$$

$$f(f^{-1}(x)) = x$$

$$5(f^{-1}(x))^3 + (f^{-1}(x)) - 7 = x$$

$$15(f^{-1}(x))^2 \cdot [f^{-1}(x)]' + [f^{-1}(x)]' = 1$$

$$[f^{-1}(x)]'\left[15(f^{-1}(x))^2 + 1\right] = 1$$

$$[f^{-1}(x)]' = \frac{1}{15(f^{-1}(x))^2 + 1}$$

$$f(f^{-1}(x)) = x$$

$$f'(f^{-1}(x)) \cdot [f^{-1}(x)]' = 1$$

$$[f^{-1}(x)]' = \frac{1}{f'(f^{-1}(x))}$$

$$y = f(x) \qquad \frac{dy}{dx} = f'(x)$$

$$x = f(y)$$

$$\frac{dx}{dy} = f'(y)$$

Q7)

$$\lim_{x\to 0} \frac{x^2}{1-\cos x} \cdot \frac{1+\cos x}{1+\cos x} =$$

$$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$$

$$= \lim_{x\to 0} \frac{x^2(1+\cos x)}{1-\cos^2 x}$$

$$\lim_{x\to 0} \frac{x}{1-\cos x} \quad \text{DNE} \quad +\infty \cdot x^{\to 0} \quad -\infty$$

$$= \lim_{x\to 0} \frac{x^2\left(1+\cos x\right)}{\sin^2 x}$$

$$= \lim_{x\to 0} \left(\frac{x}{\sin x}\right)\left(\frac{x}{\sin x}\right)\left(1+\cos x\right)$$

(19)

$$f(x) = \begin{cases} -x^4+3 & x \le 2 \\ x^2+9 & x > 2 \end{cases}$$

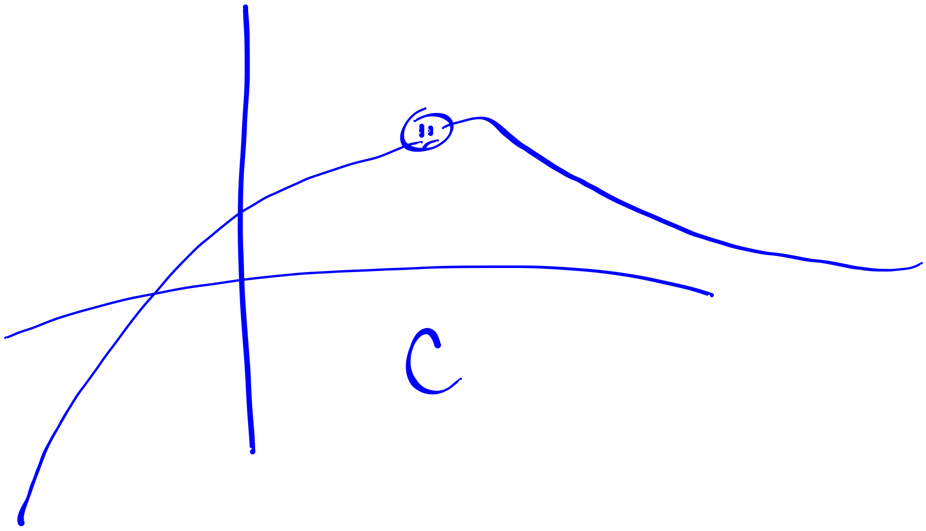
$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} -x^4+3 = -(2)^4+3$$

$$= -16+3 = -13$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} x^2+9 = 4+9 = +13$$

$$\therefore \lim_{x\to 2} f(x) \text{ dne}$$

14) a $\lim_{x \to c} f(x) = L$ but f^n is not cont



14b $f(x)$ is cont at $x = c$

but $\lim_{x \to c} f(x)$ DNE.

Not possible

Q10)

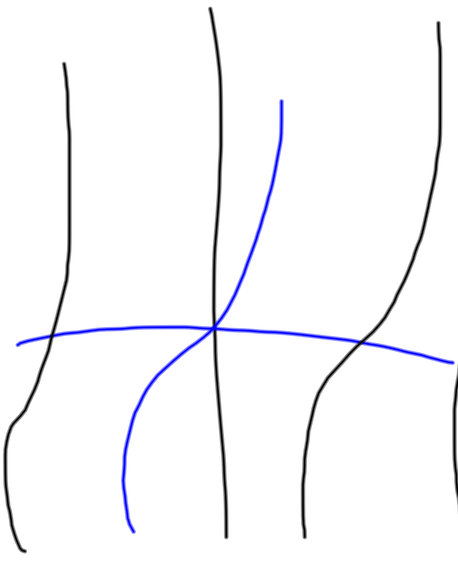
$$\lim_{x \to 0} \frac{\sqrt{x^2+4}-2}{x^2} \cdot \frac{\sqrt{x^2+4}+2}{\sqrt{x^2+4}+2}$$

$$= \lim_{x \to 0} \frac{(x^2+4)-4}{x^2(\sqrt{x^2+4}+2)} = \lim_{x \to 0} \frac{x^2}{x^2(\sqrt{x^2+4}+2)}$$

$$= \lim_{x \to 0} \frac{1}{\sqrt{x^2+4}+2} = \frac{1}{2+2} = \frac{1}{4}$$

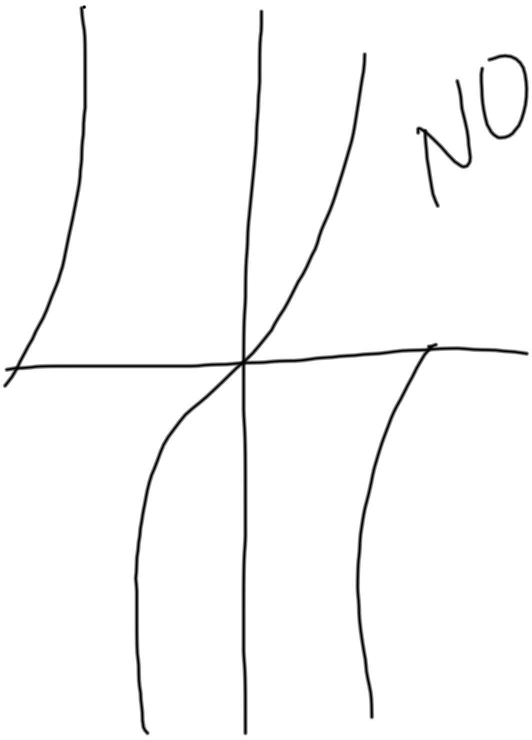
4.1

7a



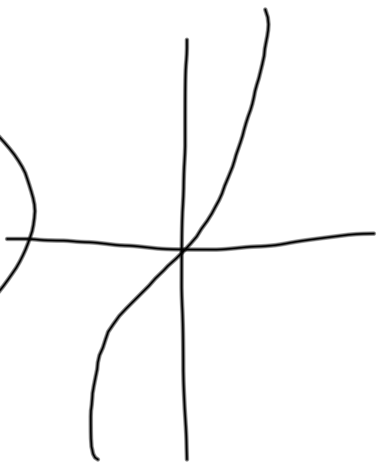
No

7b



NO

7c



Yes

4.1 11 a) $y = x^2+8x+1$

$y' = 2x+8$

Sign chart of y'

$- - - \; 0 \; + \; + + +$

-4

b) $2x^5+x^3+3x+2$

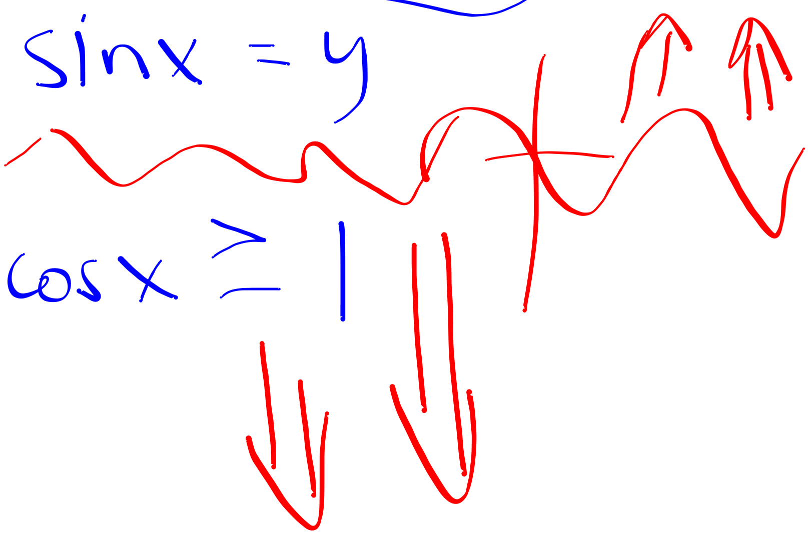
$y' = 10x^4+3x^2+3$

≥ 3

Sign chart of $10x^4+3x^2+3$

c) $2x+\sin x = y$

$y' = 2+\cos x \geq 1$



Preliminary thought - - - -

Function $f(x)$ has domain Dom
& a range of Ran
- - . -
$f^{-1}(x)$ has domain Ran
& range Dom

23)

$$f(x) = \begin{cases} \frac{5}{2} - x, & x < 2 \\ \frac{1}{x}, & x \geq 2 \end{cases}$$

$y = \frac{5}{2} - x$ | $y = \frac{1}{x}$

$x = \frac{5}{2} - y$ | $x = \frac{1}{y}$

$y = \frac{5}{2} - x$ | $y = \frac{1}{x}$

$$f^{-1}(x) = \begin{cases} \frac{5}{2} - x, & x > \frac{1}{2} \\ \frac{1}{x}, & 0 < x \leq \frac{1}{2} \end{cases}$$

4.1/27

$y = -\sqrt{3-2x}$

Dom: $(-\infty, \frac{3}{2}]$

$3-2x \geq 0$

$3 \geq 2x$

Ran: $(-\infty, 0]$

$x = -\sqrt{3-2y}$

$x^2 = (3-2y)$

$-3 \qquad -3$

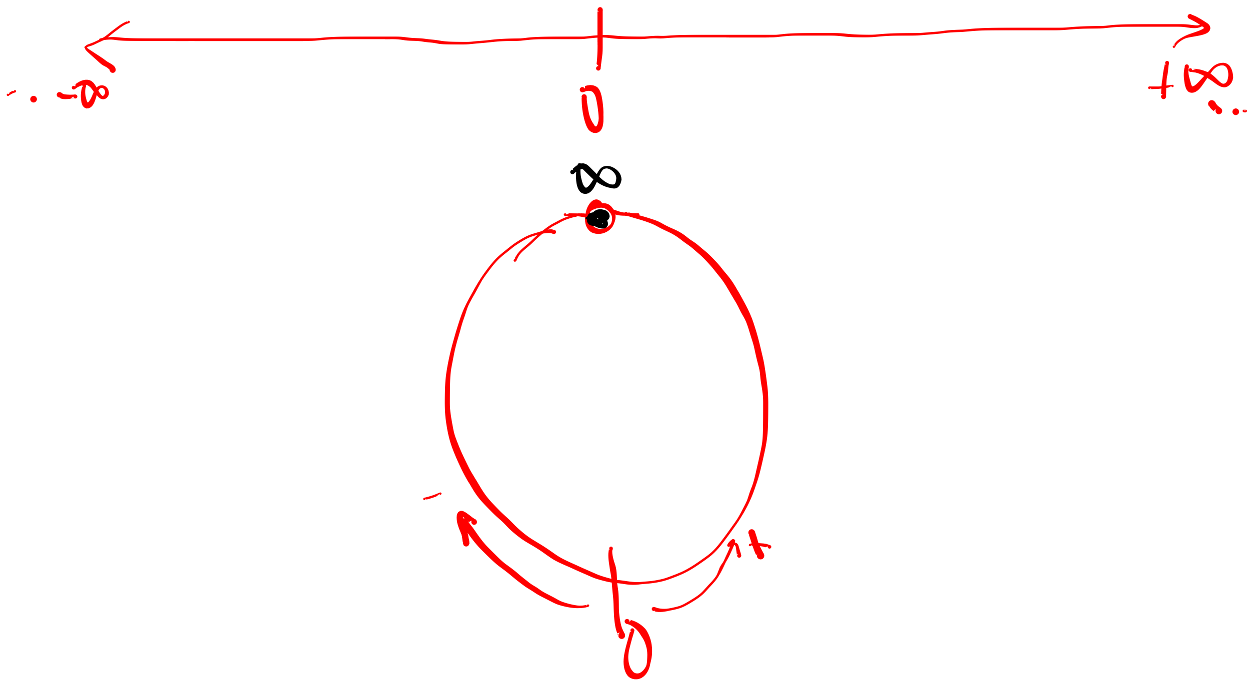
$\frac{x^2-3}{-2} = \frac{-2y}{-2}$

$\frac{x^2-3}{-2} = y$

Dom: $(-\infty, 0]$

Ran: $(-\infty, \frac{3}{2}]$

Range of $-\sqrt{3-2x}$



... -∞
0
+∞ ...
∞
-
+
0

4.2)

2^x

$\boxed{e = 2.718281828459045}$

$m = 1$

3^x

2ND Draw Tangent

$m_{\tan \text{ at } x=0} = .693$ ≈

$m_{\tan \text{ at } x=0} = 1.0986$

$(1.5)^x$

$m = .405$

5^x

$m = 1.6094$

$a^b = c$ $\log_a c = b$

$\therefore$ A logarithm is an <u>EXPONENT</u>